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Aggregable Health Inequality Indices

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Aggregable Health Inequality Indices*

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Abstract

An aggregable family of multidimensional concentration indices is characterized, in order to be consistent with a property of exogenous risk factors, *i.e.* health risks for which agents are not responsible for. The family of concentration indices (or achievement indices by duality) lies in the class of polynomial functions. Necessary and sufficient conditions are stated in order to rank two health distributions thanks to the generalized concentration curves. It is shown that the properties of mirror and symmetry are compatible with a sub-family of concentration indices being polynomial functions. A dominance criterion exists for this sub-family of indices, provided that the decision maker is an inequality lover.

Key words: Concentration, Dominance, Health Inequality, Mirror, Symmetry.

Classification JEL: D6, I1.

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1 Introduction

In the last decade, the literature on health inequality has evolved thanks to the transformation of the indices employed for the measurement of socioeconomic health inequalities, *i.e.* the health concentration indices. Those indices were initially lacking value judgments. In line with Yitzhaki's (1983) developments concerning income inequalities, socioeconomic health inequalities are now consistent with any given degree of inequality aversion.

One good feature underlying concentration indices is the decomposability property. Wagstaff, van Doorslaer and Watanabe (2003) show that socioeconomic health inequalities are decomposable into the impact of different determinants (age, consumption, etc.) by means of regression techniques. Similarly, Wagstaff (2005), following Lambert and Aronson (1993), shows that the health concentration index displays within-group, between-group, and overlapping components. Consequently, socioeconomic health inequalities are decomposable by population subgroups.

Recently, Erreygers, Clarke and Van Ourti (2012) have advocated the use of two essential properties that concentration indices have to respect: symmetry and mirror. The former postulates that permuting (upside down) the ranking of the individuals in the income distribution implies that the socioeconomic health index of inequality has the opposite sign. The latter imposes that inequality and achievement indices (achievement being the mirror concept of inequality) should actually provide opposite values.[1] In this respect, different concentration indices may be characterized with regard (or disregard) to the two aforementioned properties.

In this paper, we propose a class of concentration indices based on an *aggregation* principle rather than the usual decomposition one. This principle enables the population to be composed of heterogeneous agents. The agents are supposed to be different because they face exogenous risk factors outside their control. This is in line with existing literature concerning inequality of opportunity (see for instance Trannoy, Tubeuf, Jusot and Devaux, 2010). In particular, some health risks are supposed to be only circumstances inherent to each group of individuals. Health concentration indices, for which the main concern is looking for an association between income and health, may be blind to individuals' health statuses. Accordingly, we place the emphasis on health concentration indices that depend both on health statuses and circumstances (exogenous risk).

Following Makdissi, Sylla and Yazbeck (2013), we employ a general family of multi-dimensional concentration indices based on the rank-dependent approach. We first show, under the principles of aggregation and exogenous risk factors, that our family of concentration indices (or achievement indices by duality) are included in the class of polynomial

[1] Strictly speaking, the authors postulate that the value of the health index is the opposite of the ill health index.

functions. This result is shown to be the same for socioeconomic health indices that can be decomposable. Furthermore, it is demonstrated that this class of polynomial functions enables stochastic dominance criteria to be derived. Indeed, we provide the necessary and sufficient conditions to rank two health distributions thanks to the generalized concentration curves. Accordingly, we propose a tool – termed achievement curves – that is relevant for unambiguous ranking of health distributions composed of heterogeneous agents. Finally, it is shown that the properties of mirror and symmetry are compatible with a sub-family of concentration indices being polynomial functions. It is proven, for this sub-family, that a dominance criterion exists whenever the decision maker is inequality loving.

The paper is organized as follows. In Section 2, we uncover the class of rank-dependent multivariate socioeconomic health indices. In addition, different approaches to rank health distributions are presented: decomposition and aggregation. The family of polynomial indices based on the concept of exogenous factors is introduced in Section 3. The results concerning socioeconomic aggregable health indices being polynomial functions and the ranking by stochastic dominance are proposed in Section 4. In Section 5 the discussion about symmetry and mirror leads to multivariate generalized concentration indices not necessarily relevant with exogenous risk factors. Section 6 concludes the paper.

2 Inequality and health achievement

We follow Makdissi and Yazbeck (2014) for the counting approach regarding multidimensional health inequality indices.[2] In particular, Boolean health indicators are derived to avoid handling health indices being sensitive to an *ad hoc* scale of measurement. Let y^E be the equivalent income and p the rank of the individuals according to $y^E \in \mathbb{R}_+$, where $\mathbb{R}_+$ is the non-negative euclidean space. Let $\mathbf{H}(\mathbf{p})$ be the information related to $k = 1, \ldots, K$ health statuses (dimensions) of an individual at rank p of the income distribution, and $\Upsilon(\mathbf{H}(\mathbf{p}))$ its corresponding Boolean distribution,

$$\Upsilon(\mathbf{H}(\mathbf{p})) = \big(\iota(h_1(p)), \ldots, \iota(h_k(p)), \ldots, \iota(h_K(p))\big),$$

where

$$\iota(h_k(p)) = \begin{cases} 1, & \text{if } h_k(p) < \tau_k \\ 0, & \text{otherwise.} \end{cases}$$

The threshold τ_k for each dimension enables the counting approach to be performed. For any given individual at rank p, if the health information about dimension k falls below the threshold τ_k, the individual is considered 'poor' in that health dimension. In other terms, a health failure in dimension k is characterized by the value 1, and 0 otherwise. [3]

[2] See also Alkire and Foster (2011) for the counting approach in the case of poverty measurement.

[3] As pointed out by Makdissi and Yazbeck (2014), $\Upsilon(\mathbf{H}(\mathbf{p}))$ is robust to any given monotonic transformation of $\mathbf{H}$ avoiding a particular view being imposed on the transformation of health status.

Based on this health information, a general aggregator $\phi : [0,1]^K \longrightarrow \mathbb{R}_+$ across health dimensions has to be imposed in order to rank health distributions $\mathbf{H}$. Any suitable $\phi(\cdot)$ aggregator may be employed, for instance Makdissi and Yazbeck (2014) propose:

$$\phi(\mathbf{H}(\mathbf{p})) = \frac{K - \Upsilon(\mathbf{H}(\mathbf{p}))\Theta'}{K},$$

where Θ' is an n-dimensional column vector of weights such that the ℓ_1 norm is $\|\Theta\|_1 = K$. The index $\phi(\mathbf{H}(\mathbf{p}))$ is a normalized mean representing the health achievement for an individual at rank p. Accordingly, the social rank-dependent achievement health index is invariant to any monotonic transformation of the health information $\mathbf{H}(\mathbf{p})$:

$$A'(\mathbf{H}) = \int_0^1 v(p)\phi(\mathbf{H}(\mathbf{p}))dp. \tag{A0}$$

The weight function $v : [0,1] \longrightarrow [0,1]$, such that $\int_0^1 v(p)dp = 1$, is the usual one related to the social planner's preferences, which may impose more or less weight at the tails of the transformed distribution $\phi(\mathbf{H})$, see Aaberge (2009). This health index has been extensively used in the one-dimensional literature, because it enables the well-known concentration index to be retrieved as a special case – see for instance Wagstaff *et al.* (2003).

Several tools may be employed to deal with heterogeneous agents. They rely on, respectively, the decomposition approach and the aggregation approach. In what follows, we will use the terms *group* and *type* with the same meaning.

2.1 Decomposition

Although our main findings are concerned with the aggregation approach and the stochastic dominance approach, it is of interest to remember the proportion in relevant literature of inequality decompositions in which the concentration index represents the building-block pattern.

Wagstaff *et al.* (2003) explain that the socioeconomic statuses of an individual can be decomposed into the impact of various factors by means of regression techniques, and accordingly, socioeconomic health inequalities are decomposable into the impact of different determinants such as child age, household consumption, etc. On the other hand, in order to exhibit the heterogeneity of the population, Wagstaff (2005) makes use of the decomposition technique initiated by Lambert and Aronson (1993), in such a way that the concentration index displays a within-group inequality term, a between-group inequality term, and finally an overlapping (transvariational) component.[4] This yields the ability to gauge the variation of the inequalities within groups and between groups.

[4]See Gini (1916) and Dagum (1959). The transvariation means that the inequality between individuals, computed on the basis of income gaps, has a reverse sign compared with that of the mean income gap of the group they belong to.

Makdissi *et al.* (2013) generalize this decomposition method in a multivariate context using the index (A0). They provide a weight function $v(p) = \nu(1-p)^{\nu-1}$ that captures a wide spectrum of social planner behaviors (see *e.g.* Yitzhaki (1983) for the Gini index), so that the multidimensional achievement index is refined as follows:

$$A'_\nu(\mathbf{H}) = \int_0^1 \nu(1-p)^{\nu-1}\phi(\mathbf{H}(p))dp,\ \nu > 1.$$

By duality, the generalized socioeconomic health index of inequality is:

$$I'_\nu(\mathbf{H}) = 1 - \frac{A'_\nu(\mathbf{H})}{\mu_\phi},\ \nu > 1,$$

with $\mu_\phi = \int_0^1 \phi(\mathbf{H}(\mathbf{p}))dp$. If $\nu > 2$ the index displays health inequality aversion, whereas health inequality loving is obtained whenever $\nu \in (1,2)$. If $\nu = 2$ the concentration index is deduced.[5] As a consequence, the socioeconomic health index of inequality outlines within-group, between-group and transvariational indices:

$$I'_\nu(\mathbf{H}) = I_{W,\nu}(\mathbf{H}) + I_{B,\nu}(\mathbf{H}) + I_{T,\nu}(\mathbf{H}),\ \nu > 1.$$

Another option to deal with the decomposition technique is the use of stochastic dominance. Makdissi and Mussard (2008a) show that the usual concentration curves can be generalized into s-curves ($\mathcal{C}^s$) in order to capture more value judgments and allowing for more redistribution toward the poor insofar as the value of s is the highest possible. For each value of s, the concentration curves are decomposable into within-group $\mathcal{C}^s_W$, between-group $\mathcal{C}^s_B$, and interaction (transvariational) curves $\mathcal{C}^s_T$, so that: $\mathcal{C}^s = \mathcal{C}^s_W + \mathcal{C}^s_B + \mathcal{C}^s_T$.[6] As far as s increases, more weight is put on the left-hand side of the socioeconomic health distribution, implying that the social planner is more averse to health inequalities. In this respect, some dominance results can be derived. For instance, decision makers only interested in within-group inequalities, admittedly between two socioeconomic distributions $\mathbf{H}_1$ and $\mathbf{H}_2$, will rank those two situations according to the within-group dominance criterion: If the within-group s-curve of $\mathbf{H}_1$ lies nowhere below that of $\mathbf{H}_2$ *i.e.* $\mathcal{C}^s_W(\mathbf{H}_1) \geqslant \mathcal{C}^s_W(\mathbf{H}_2)$ over the entire percentile space, then the decision maker will prefer distribution $\mathbf{H}_1$. This preference is possible even if the between-group inequalities are higher in $\mathbf{H}_1$ such that $\mathcal{C}^s_B(\mathbf{H}_1) \leqslant \mathcal{C}^s_B(\mathbf{H}_2)$. Thereby, dominance conditions may be restricted to within-group, between-group or transvariational dominance, and in some cases, the combination of at least two dominance conditions.

In the decomposition methods described above, although the point of view of the decision maker is variable thanks to the parameters ν or s, there is only one attitude toward inequality, that is, only one weight $v(p)$ for the entire population with regard to the dominance order. The second approach below deals with multiple weights.

[5]See Wagstaff (2002) for the link between achievement and concentration indices in the univariate setting.

[6]See Wagstaff and van Doorslaer (2004) for an application to socioeconomic health inequalities in Canada for the standard case $s = 2$.

2.2 Aggregation

The other alternative to deal with the heterogeneity of agents is to use a rank-dependent achievement health index with variable weights according to the group the individual belongs to:

$$\begin{aligned} A(\mathbf{H}) &= \sum_{i=1}^{n} \kappa_i \int_0^1 v_i(p)\phi_i(\mathbf{H}(\mathbf{p}))dp \\ &=: \sum_{i=1}^{n} \kappa_i A_i(\mathbf{H}), \end{aligned} \tag{A1}$$

where κ_i is the population share of type i people[7], where $v_i(p)$ are weight functions that reflect the preferences of the individuals of type i at rank p, and where ϕ_i is the function ϕ itself applied to type i individuals only.[8] The index (A1) is based on aggregated concentration indices with different weighting schemes $v_i : [0,1] \longrightarrow [0,1]$ for all $i = 1,\ldots,n$. By duality, aggregable indices of socioeconomic health inequality are:

$$I(\mathbf{H}) = 1 - \frac{A(\mathbf{H})}{\mu_\phi},$$

where $\mu_\phi = \int_0^1 \phi(\mathbf{H}(p))$ is the overall mean of health information. Let $\mu_{\phi_i} \equiv \mu_{\phi_i(\mathbf{H})} := \int_0^1 \phi_i(\mathbf{H}(\mathbf{p}))dp$ be the average of health information of type i individuals. Defining the weights $\theta_i := \kappa_i \mu_{\phi_i}/\mu_\phi$ that add up to unity, we obtain:

$$\begin{aligned} I(\mathbf{H}) = 1 - \frac{\sum_{i=1}^{n} \kappa_i A_i(\mathbf{H})}{\mu_\phi} &= \sum_{i=1}^{n} \theta_i \left(1 - \frac{A_i(\mathbf{H})}{\mu_{\phi_i}}\right) \\ &=: \sum_{i=1}^{n} \theta_i I_i(\mathbf{H}), \end{aligned} \tag{I1}$$

where $I_i(\mathbf{H})$ is the socioeconomic health inequality index of type i people, and where (I1) is the equivalent assumption of (A1) that defines the class of aggregable socioeconomic health inequality indices.

There are many different explanations for the heterogeneity of agents. In this paper, we are interest in the heterogeneity of individuals when they face different risk factors that can affect their health status. In the following sections, we suggest the possibility of using the aggregation approach in order to link the measurement of socioeconomic health inequality with some unavoidable risk factors that agents cannot control.

[7] If the number of type i individuals is denoted N_i such that $N = \sum_i N_i$, then $\kappa_i = N_i/N$.

[8] For simplicity we can set $\phi_i \equiv \phi$. Alternatively, it is be possible to set,

$$\phi_i(\mathbf{H}(\mathbf{p})) = \frac{K - \Upsilon_i(\mathbf{H}(\mathbf{p}))\Theta_i'}{K},$$

where Υ_i captures the health failure of type i people, so in this case the identification function would be specific to each group in the same manner than the weight function Θ_i'.

3 Exogenous risk factors and polynomial indices

The employ of different weighting schemes in (A1) is of interest in order to lay the emphasis on particular groups of the population. We are interested in socioeconomic health inequality when the population is partitioned according to any given exogenous risk factor. For example, the occupational status might be one cause of the variation in an individual's health status, because the occupational status is related to health *via* two channels. First, every job has its own specific activities and rewards, which can influence health, such as being physically hazardous or involving psychologically stressful working conditions (House, Wells, Landerman, McMichael and Kaplan (1980); Karasek, Baker, Marxer, Ahlbom and Theorell (1981)). The second channel is an indirect one as each job can have a negative influence on lifestyle behavior including drinking, smoking and obesity (Sorenson, Pirie, Folsom, Luepker, Jacobs and Gillum (1985); House, Stretcher, Metzner and Robbins (1986)). There can also be a positive influence on lifestyle, because the income gained from a specific activity can provide the funds to, among other things, purchase medical care, healthy food, and the possibility to choose a safe living environment.

When someone chooses a particular job, we might think they would evaluate the associated health risks. However, their choice is not always the result of free will, but of the unique possibilities given the specific circumstances. Literature concerning inequality of opportunity makes a distinction between *circumstances*, which are beyond an individual's control and *efforts*, which are under their control. Several papers try to measure inequality of opportunity in health. Gakidou, Murray and Frenk (1999) make the distinction between unavoidable factors (chance, genes at birth) and choices (addictive habits) to analyze the distribution of health expectancy. Rosa Dias (2009) implements stochastic dominance tests to detect inequality of opportunity in the conditional distributions of self-assessed health in adulthood. In another paper, Rosa Dias (2010) considers unobserved heterogeneity in order to measure inequality of opportunity in health. Along the same line of research, Trannoy *et al.* (2010) investigate inequality of opportunity in health by analyzing the role of circumstances during childhood such as family and social backgrounds. Other branches of papers pay specific attention to the way circumstances and efforts are correlated. In particular, they are interested in discovering if it empirically matters which normative way of treating these two notions is used. See for instance Jusot, Tubeuf, and Trannoy (2013), and Bricard, Jusot, Trannoy and Tubeuf (2013).

Le Clainche and Wittwer (2015) analyze the impact of risky behaviors and find, on the basis of samples from four countries, that students would accept having to pay for the health costs related to their risky choices under their responsibility. By contrast, in our paper, we look for the differences in health risks that are exogenous factors we can interpret, as explained before, as differences in circumstances. The health risks are supposed to be outside of control, *i.e.* they are only circumstances inherent to each group of individuals. In our example, the occupational status reflects the outcomes of educational attainments. At the same time, the individual's educational level is not only the consequence of their efforts and choices, but it is also constrained by circumstances. In other words, education is correlated with features of the external environmental or with cognitive abilities. Accordingly, the occupational status is strongly constrained by childhood circumstances. In this context, individuals do not have the possibility to control for their professional risks, which are considered exogenous and differ from one profession to another. The example of professional risks will be used later in order to understand the differences between the agents in exogenous factors.

For this purpose, health risk is embodied by the weights defined by Yaari (1987) in his seminal dual approach. The intensity of the weights describes the relative importance of the risks associated with the different groups of individuals. These are defined as follows (see Jenkins and Lambert (1993) for the related notion of differences in needs with regard to welfare economics).

Definition 3.1 – **Differences in exogenous risk factors:** *The functions* $v_i^{(0)}(\cdot) := v_i(\cdot)$ *for all* $i = 1, \ldots, n$ *and for all* $p \in [0, 1]$ *are such that:*

(i) $v_1^{(\ell)}(p) \geqslant \cdots \geqslant v_i^{(\ell)}(p) \geqslant \cdots \geqslant v_n^{(\ell)}(p) \geqslant 0,\ \ell = 0$;

(ii) $0 \leqslant (-1)^\ell v_1^{(\ell)}(p) \leqslant \cdots \leqslant (-1)^\ell v_i^{(\ell)}(p) \leqslant \cdots \leqslant (-1)^\ell v_n^{(\ell)}(p),\ \forall \ell = 1, \ldots, s-1.$

This definition shows two properties as mentioned by Brunori, Palmisano and Peragine (2014) in the case of income taxation.

(i) *Vertical equity.* The weights are such that $v_{i+1}(p) \leqslant v_i(p)$ for all $p \in [0, 1]$. This property states that the individual types are ranked according to an exogenous risk factor. For example, professional categories i are ranked according to the risk they imply. The greater the risk, the higher the weight that is assigned to the employment category by the social planner.

(ii) *Horizontal equity.* The weights are such that $(-1)^\ell v_i^{(\ell)}(p) \leqslant (-1)^\ell v_{i+1}^{(\ell)}(p)$. In other words, within each type, health transfers toward people with poor health are permitted. However, these transfers are of greater importance for groups with a higher risk level. More precisely, the risks are defined by the importance of the weight variations that directly affect the transformed distributions $\phi_i(\mathbf{H})$ of each type i. In the riskiest group $i = 1$, an increase in at least one achievement health dimension is more valuable than in group $i = 2$, and so on. It is notable that, in our framework, the cardinalization of $v_i^{(\ell)}(p)$ is unnecessary. Only the ranking of the different weights $v_i^{(\ell)}(p)$ matters to obtain an unambiguous ranking of health distributions, as is shown in Theorem 4.1 below.

Specifically, for each individual type i, the signs of the successive derivatives of $v_i(\cdot)$ yield the generalized positional transfer sensitivity principle. This is defined and generalized by Aaberge (2009), Makdissi and Mussard (2008a), and suitably employed in health literature by Makdissi and Yazbeck (2014) for any given order of dominance s.[9]

• [$s=1$]: The first-order positional transfer principle, embodied by $v_i(\cdot)^{(0)} = v_i(\cdot) \geqslant 0$, is related to Pen's parade whenever the symmetry of the social health achievement function $A(\mathbf{H})$ is ensured.[10] It states that a distribution $\tilde{\mathbf{H}}$ is issued from $\mathbf{H}$ by the improvement of one health status k of an individual at rank p_0, *ceteris paribus*. Pen's parade is respected if, $A(\tilde{\mathbf{H}}) \geqslant A(\mathbf{H})$.

• [$s=2$]: The second-order positional transfer principle, embodied by $v_i^{(0)}(\cdot) \geqslant 0$ and $v_i^{(1)}(\cdot) \leqslant 0$, is the transfer sensitivity property (known as the progressive Pigou-Dalton transfer principle in a utilitarian framework). It is defined based on the variation of the health achievement of an individual at rank p. It postulates that a (progressive) health transfer of amount $\delta > 0$ from an individual at rank $p_{j,i}$ to another individual at rank $p_{j',i}$, where j and j' are of the same individual type i such that $p_{j,i} = p_{j',i} + \gamma$ with $\gamma > 0$, yields *ceteris paribus*, an health achievement distribution $\tilde{\mathbf{H}}$. The second-order positional transfer principle is respected if:

$$\underset{\gamma,(j,j')}{\overset{1}{\Delta}} A(\mathbf{H}) := A(\tilde{\mathbf{H}}) - A(\mathbf{H}) \geqslant 0 \ .$$

• [$s=3$]: The third-order positional transfer principle, embodied by $v_i^{(0)}(\cdot) \geqslant 0$, $v_i^{(1)}(\cdot) \leqslant 0$ and $v_i^{(2)}(\cdot) \geqslant 0$ (Kolm's transfer principle in the utilitarian layout) postulates that a transfer sensitivity is more valuable to another, the lower it appears on the ranks of the distribution of health achievements. Let $j < j' < l < l'$ with $j'-j = l'-l = \gamma > 0$ such that an health achievement distribution $\tilde{\mathbf{H}}$ is issued from a progressive transfer sensitivity of health between j and j' and a regressive[11] one between l' and l, *ceteris paribus*. The third-order positional transfer principle is respected if,

$$\underset{\gamma,(j,l,j',l')}{\overset{2}{\Delta}} A(\mathbf{H}) := \underset{\gamma,(j,j')}{\overset{1}{\Delta}} A(\mathbf{H}) \ - \ \underset{\gamma,(l,l')}{\overset{1}{\Delta}} A(\mathbf{H}) \geqslant 0 \ .$$

• [order s]: Positional transfers principle of order s recursively yields a health achievement distribution $\tilde{\mathbf{H}}$ by combining a positional transfer principle of order $s-1$ at the lower part of the distribution with a regressive positional transfer principle of order $s-1$ at the upper part of the distribution. The sth positional transfer principle is respected if,

$$\underset{\gamma,(j,l,\ldots,k',h')}{\overset{s}{\Delta}} A(\mathbf{H}) = \underset{\gamma,(j,\ldots,h')}{\overset{s-1}{\Delta}} A(\mathbf{H}) \ - \ \underset{\gamma,(l,\ldots,k')}{\overset{s-1}{\Delta}} A(\mathbf{H}) > 0 \ ; \tag{A2}$$

$$\textit{and} \quad \underset{\gamma,(j,l,\ldots,h'+1,k'+1)}{\overset{s+1}{\Delta}} A(\mathbf{H}) = 0. \tag{A3}$$

[9] Strictly speaking, Aaberge (2009) defined the downward positional principle and the upward one. In what follows, we use the downward principle only, which states that the decision maker prefers the poor to become richer ($v_i^{(3)}(\cdot) \leqslant 0$). Note that the analysis could have been done with the upward principle that states that the social planner prefers the rich to become poorer ($v_i^{(3)}(\cdot) \geqslant 0$).

[10] Note that the symmetry of $A(\mathbf{H})$ is ensured whenever ϕ is symmetric in the different health categories k.

[11] Regressive means that the transfer occurs from a lower-rank individual to a higher-rank one.

A social achievement index $A(\mathbf{H})$ satisfies the first-order principle if, and only if, $v_i(p) \geqslant 0$ for all $p \in [0,1]$ and for all $i = 1, \ldots, n$. It can be noted that condition (A3) is not necessary for dominance purposes, but it will be of interest in the characterization of the indices $A(\mathbf{H})$ below. It postulates that whenever the sth principle of positional transfer sensitivity is respected, then the $s+1$th principle is neutral in the sense that no health variations are recorded.

It is also possible to impose more structure to the achievement health indices by including the sth order positional transfer principle, by defining the following set:

$$\Omega^s := \left\{ A(\mathbf{H}) \in \mathbb{R} \;\middle|\; \begin{array}{l} v_i^{(\ell)} \text{ is continuous and } s\text{-time differentiable everywhere over } [0,1] \\ (-1)^\ell v_i^{(\ell)}(p) \geqslant 0 \ \forall p \in [0,1] \ ; \forall i = 1,2,\ldots,n \ ; \forall \ell = 1,\ldots,s-1 \\ v_i^{(\ell)}(1) = 0, \ \forall i = 1,2,\ldots,n \ ; \forall \ell = 1,\ldots,s-1. \end{array} \right\}$$

A social achievement health index $A(\mathbf{H})$ satisfies the sth-order positional transfer principle if, and only if, $A(\mathbf{H}) \in \Omega^s, \ \forall s \in \{1,2,3\ldots\}$ with $\Omega^s \subset \cdots \subset \Omega^3 \subset \Omega^2 \subset \Omega^1$.

Lemma 3.1 *For all $A(\mathbf{H}) \in \Omega^s$ such that $s \in \{1,2,3,\ldots\}$ respecting (A1), (A2), and (A3), the aggregable achievement index $A(\mathbf{H})$ and the weight functions $v_i(p)$ are polynomial of degrees at most s and $s-1$, respectively.*

Proof:
Let us consider health variations in group i, such that (A2) and (A3) apply for group i only. Hence by Aczél (1966, p.130) the function $\int_0^1 v_i(p)\phi_i(\mathbf{H}(\mathbf{p}))dp$ is a polynomial of degrees at most s. Applying the same reasoning to the other groups entails that $\sum_{i=1}^n \int_0^1 v_i(p)\phi_i(\mathbf{H}(\mathbf{p}))dp$ is also a polynomial of degrees at most s. Note that by construction $\phi_i(\mathbf{H}(\mathbf{p}))$ is a linear transformation of the Boolean function $\Upsilon(\mathbf{H}(\mathbf{p}))$, the images of which are independent of p. Consequently, $v_i(p)$ is a polynomial of degrees at most $s-1$. ■

This result allows one to restrict the class of aggregable achievement indices to polynomial functions. It is of interest to note that the same results apply for decomposable measures (A0). If we define Ω'^s the same set as Ω^s for $A'(\mathbf{H})$ indices, then we obtain the following.

Lemma 3.2 *For all $A'(\mathbf{H}) \in \Omega'^s$ such that $s \in \{1,2,3,\ldots\}$ respecting (A0), (A2), and (A3), the achievement index $A'(\mathbf{H})$ and the weight function $v(p)$ are polynomial of degrees at most s and $s-1$, respectively.*

Proof:
Mutatis mutandis the proof of Lemma 3.1. ■

From Lemma 3.2, the class of socioeconomic health inequality indices proposed by Makdissi and Yazbeck (2014),

$$I'(\mathbf{H}) = 1 - \frac{1}{\mu_\phi}\int_0^1 v(p)\phi(\mathbf{H}(\mathbf{p}))dp,\ \nu > 1,$$

reduces to a polynomial of degrees at most s. Hence their choice of imposing $v(p) = \nu(1-p)^{\nu-1}$, where $\nu > 1$ represents the aversion toward health inequality, is perfectly relevant since it is a polynomial of degrees $s-1 = \nu-1$. Lemma 3.1 states that the Pigou-Dalton transfer principle is respected ($s = 2$) whenever the degree of the polynomial is at most $s-1 = 1$. For higher orders, $v(p) = s(1-p)^{s-1}$ such that the degree of the polynomial cannot be lower than $s-1$ otherwise the s-th positional transfer principle would be violated (in the case of the Pigou-Dalton principle, $s-1 = 1$ and it cannot be that $s-1 < 1$). Thereby, for the family of socioeconomic health inequality indices $I'_\nu(\mathbf{H})$, the degree of the polynomial must be exactly $s-1 = \nu-1$, and we retrieve the standard concentration index when $s = 2$. As mentioned by Erreygers *et al.* (2012, p.264), concentrations indices are not the only ones that incorporate distributional sensitivity, however they are often employed in practice. Actually, Lemma 3.2 operates this way, because imposing few assumptions on the structure of the index implies the recourse to polynomial functions, which lead to a functional form very close to concentration indices.

The same property holds true for aggregable socioeconomic health indices of inequality defined by:

$$I(\mathbf{H}) = 1 - \frac{A(\mathbf{H})}{\mu_\phi}.$$

Taking a weight function $v_i(p)$ with regard to the result of Lemma 3.1 such that $v_i(p) = \alpha_i s(1-p)^{s-1}$ for $\alpha_i > 0$ and for all $i = 1, \ldots, n$, then $I(\mathbf{H})$ is nothing other than a weighted mean of generalized concentrations indices:

$$I(\mathbf{H}) = \sum_{i=1}^{n}\theta_i\left[1 - \frac{1}{\mu_{\phi_i}}\int_0^1 \alpha_i s(1-p)^{s-1}\phi_i(\mathbf{H}(\mathbf{p}))dp\right],$$

with $s \geqslant 2$ to achieve inequality aversion, $s \in (1,2)$ to get equality aversion, and finally $\alpha_1 \geqslant \cdots \geqslant \alpha_n$ to ensure the differences in risks to be captured (vertical equity (i)).

If some transfers principles are permitted in some groups, but not in others, it is possible to invoke (A1), (A2), and (A3) separately for each $A_i(\mathbf{H})$. From this perspective, health transfers are type i specific so that the socioeconomic health index of inequality would become:

$$I_1(\mathbf{H}) = \sum_{i=1}^{n}\theta_i\left[1 - \frac{1}{\mu_{\phi_i}}\int_0^1 s_i(1-p)^{s_i-1}\phi_i(\mathbf{H}(\mathbf{p}))dp\right],$$

with $s_i \geqslant 2$ to capture inequality aversion in group i and $s_i \in (1,2)$ for equality aversion.

4 Main results

We now turn to the specification of our main results: Ranking health distributions on the basis of either aggregable achievement indices (A1) or aggregable socioeconomic health inequality indices (I1) with a sequential dominance criterion.

As seen above, different indices can be designed with regard to the aggregation rule (A1). Since agents are supposed to be heterogeneous by facing different exogenous risks factors, aggregable health indices can be shown to be also in line with literature concerning sequential stochastic dominance. From this perspective, the advantage of using this approach is to rank health distributions independently of the calibration of either the achievement index or the inequality index. The ranking depends on the comparison of only concentration curves, specifically achievement curves. The generalized achievement curve of order 1 related to individuals of type i is $GAC^1_{\mathbf{H},i}(p) := (K - \Upsilon_i(\mathbf{H}(\mathbf{p}))\Theta')/K$. The generalized achievement curve of order $s \in \{2, 3, \ldots\}$ is:

$$GAC^s_{\mathbf{H},i}(p) := \int_0^p GAC^{s-1}_{\mathbf{H},i}(u)\, du.$$

For example, the generalized achievement curve of order 2 for group i ($GAC^2_{\mathbf{H},i}(p)$) provides the cumulative health achievement held by the individuals (ranked p and below) in group i. Accordingly, the following result is obtained.

Theorem 4.1 *Under Definition 3.1 of exogenous risk factors, for all health achievement indices $A(\mathbf{H}) \in \Omega^s$, where $s \in \{1, 2, 3, \ldots\}$, respecting (A1) and (A2), the two following statements are equivalent:*

(i) $A(\mathbf{H}) \geqslant A(\tilde{\mathbf{H}})$

(ii) $\sum_{i=1}^{l} \kappa_i \left[GAC^s_{\mathbf{H},i}(p) - GAC^s_{\tilde{\mathbf{H}},j}(p) \right] \geqslant 0,\ \forall l \in \{1, \ldots, n\},\ \forall p \in [0, 1].$

Proof:

See the appendix. ■

Theorem 4.1 states that when comparing distributions $\tilde{\mathbf{H}}$ and $\mathbf{H}$, the dominance of the generalized achievement curves does not have to be checked for all groups i. The dominance condition (ii) between $\mathbf{H}$ and $\tilde{\mathbf{H}}$ has to be verified (for sure) for the first group. Indeed, the proportion of individuals with worse circumstances (type $i = 1$) in $\mathbf{H}$ must be lower than in $\tilde{\mathbf{H}}$ for each percentile p. After checking the first group, we can check the others, however, the previous groups are taken into account. Hence, the dominance condition is weaker than when checking for the dominance group by group.

The same result can be derived for inequality indices of socioeconomic health satisfying the aggregation condition (I1). For this purpose, since $I(\mathbf{H})$ is obtained by construction from $A(\mathbf{H})$, the class of socioeconomic health inequality is:

$$\Xi^s := \left\{ I(\mathbf{H}) \in \mathbb{R} \;\middle|\; \begin{array}{l} v_i^{(\ell)} \text{ is continuous and } s\text{-time differentiable everywhere over } [0,1] \\ (-1)^\ell v_i^{(\ell)}(p) \geqslant 0 \; \forall p \in [0,1] \; ; \forall i = 1,2,\ldots,n \; ; \forall \ell = 1,\ldots,s-1 \\ v_i^{(\ell)}(1) = 0, \;\; \forall i = 1,2,\ldots,n \; ; \forall \ell = 1,\ldots,s-1. \end{array} \right\} \tag{I2}$$

The condition (I2) encompasses an equivalent property of (A2), *i.e.* health transfers within health type i enable the inequality to be reduced. Indices in Ξ^s respect the s-th principle of positional transfer sensitivity, so that the health inequality decreases after the application of such a transfer. In order to derive a dominance rule, the achievement curve of order 1 related to individuals of type i is defined as $AC^1_{\mathbf{H},i}(p) := \frac{K - \Upsilon_i(\mathbf{H}(\mathbf{p}))\Theta'}{K\mu_{\phi_i}}$. The achievement curve of order $s \in \{2,3,\ldots\}$ is:

$$AC^s_{\mathbf{H},i}(p) := \int_0^p AC^{s-1}_{\mathbf{H},i}(u)\, du.$$

The achievement curve of order 2 for group i ($AC^2_{\mathbf{H},i}(p)$) yields the cumulative health achievement proportion held by the individuals in group i (ranked p and below). Accordingly, the dual result of Theorem 4.1 is found.

Theorem 4.2 *Under Definition 3.1 of exogenous risk factors, for all aggregable socioeconomic health inequality indices* $I(\mathbf{H}) \in \Xi^s$, *where* $s \in \{1,2,3,\ldots\}$, *respecting* (I1) *and* (I2), *the two following statements are equivalent:*

(i) $I(\tilde{\mathbf{H}}) \leqslant I(\mathbf{H})$

(ii) $\sum_{i=1}^{l} \theta_i \left[AC^s_{\mathbf{H},i}(p) - AC^s_{\tilde{\mathbf{H}},j}(p) \right] \leqslant 0, \;\; \forall l \in \{1,\ldots,n\}, \;\; \forall p \in [0,1].$

Proof:

See the appendix. ■

5 Symmetry and mirror: Discussions and results

The dominance approach relies on fewer assumptions compared with the aggregation approach, because only the ranking of the functions $v_i(p)$ and the sign of their derivatives matter with regard to ascertaining whether the groups face bad or good circumstances (for example, lower or higher risk in their professional activity). It is notable that, for dominance purposes, assumption (A3) is relaxed since it yields only an upper bound of degree of the polynomial function. The dominance approach is therefore more general and requires fewer assumptions, nevertheless the dominance criterion remains silent whenever the achievement curves cross. In this case, the ranking between $\mathbf{H}$ and $\tilde{\mathbf{H}}$ in terms of achievement or health inequality is not possible unless a higher-order dominance arises.

The three approaches may all suffer due to disrespecting two very general properties: symmetry and mirror – see Erreygers *et al.* (2012). The symmetry property is one of the cornerstones of literature concerning income inequality measurement. It postulates that the decision maker in charge of aggregating the preferences is behind the veil of ignorance, thereby the labeling of individuals does not imply any variation of the inequality index. Specifically, from Erreygers *et al.* (2012), if individuals are ranked in the reverse order to their initial ranking in the income distribution, then the index has to produce the opposite value to the initial inequality in society (see also the close concept of consistency property introduced by Lambert and Zheng, 2011). On the other hand, the mirror property postulates that, since achievement and inequality are mirror concepts, then the index defined either on achievement, $\phi(\mathbf{H}(p))$, or deprivation, $1-\phi(\mathbf{H}(p))$, must provide opposite values. Consequently, it is necessary to find socioeconomic health inequality indices that are consistent with exogenous risk factors, symmetry and mirror properties.

Because the aggregable indices $I(\mathbf{H})$ or $I_1(\mathbf{H})$ are useful for dominance purposes, it would be interesting to test them with regard to the two above-mentioned properties. First, it can be noted that Erreygers *et al.* (2012) restrict their approach to the class of Merhan's concentration indices $\mathcal{M}$, which we rewrite in a multidimensional setting with the aggregation principle as follows:[12]

$$M(\mathbf{H}) = \sum_{i=1}^{n} \theta_i \; f(\mu_{\phi_i}) \int_0^1 w_i(p,s)\phi_i(\mathbf{H}(p)) =: \sum_{i=1}^{n} \theta_i M_i(\mathbf{H}),$$

where $f(\mu_{\phi_i})$ is a normalization function and where $w_i(p,s)$ is the weight function associated with the inequality aversion s of group i. Rewriting $I(\mathbf{H})$ as:

$$I(\mathbf{H}) = \sum_{i=1}^{n} \theta_i \left[\frac{1}{\mu_{\phi_i}} \int_0^1 \left(1-\alpha_i s(1-p)^{s-1}\right) \phi_i(\mathbf{H}(\mathbf{p}))dp \right] = \sum_{i=1}^{n} \theta_i I_i(\mathbf{H}),$$

where $\frac{1}{\mu_{\phi_i}} = f(\mu_{\phi_i})$. Setting $w_i(p,s) = 1-\alpha_i s(1-p)^{s-1}$ such that $\alpha_i s(1-p)^{s-1} = v_i(p)$, we deduce that the sth positional transfer principle is respected, that is $I(\mathbf{H}) \in \{\mathcal{M} \cap \Xi^s\}_{s\geqslant 2}$. As a consequence, the results of Erreygers *et al.* (2012) concerning symmetry and mirror may also apply to $I(\mathbf{H})$. Let us rewrite the properties of symmetry and mirror in a multivariate setting for the indices being in the class $\mathcal{M}$.

Definition 5.1 Symmetry (SYM) – *A socioeconomic health inequality index $M(\mathbf{H}) \in \mathcal{M}$ is symmetric, if for $\mathbf{G}(\mathbf{p}) := \mathbf{H}(1-\mathbf{p})$, then $M(\mathbf{G}) = -M(\mathbf{H})$.*

The idea underlying symmetry is that a negative value indicates that the health distribution $\mathbf{H}$ is pro-poor regarding health, and conversely.

Definition 5.2 Mirror (MIR) – *A socioeconomic health inequality index $M(\mathbf{H}) \in \mathcal{M}$ respects the mirror principle, if for $\phi(\mathbf{G}(\mathbf{p})) := 1-\phi(\mathbf{H}(\mathbf{p}))$, then $M(\mathbf{G}) = -M(\mathbf{H})$.*

[12] We use $f(\mu_\phi)$ instead of $f(\mu_\phi, s)$.

Mirror postulates that an index must not be sensitive to inequality or achievement views, which are actually dual concepts.

The results about the respect of (MIR) and (SYM) for the class of indices $\mathcal{M}$ are the following.

Proposition 5.1 *An aggregable socioeconomic health inequality index such that* $M(\mathbf{H}) \in \mathcal{M}$ *with* $s \in \{1, 2, 3, \ldots, \}$ *and* $\int_0^1 w_i(p, s)dp = 0$ *for all* $i = 1, \ldots, n$*, displays the following properties:*

(i) $M(\mathbf{H})$ *respects* (SYM)*, if and only if,* $w_i(p, s) = -w_i(1-p, s)$ *for all* $i = 1, \ldots, n$ *;*

(ii) $M(\mathbf{H})$ *respects* (MIR)*, if and only if, the function* $f(\cdot)$ *is independent of* μ_{ϕ_i}.

Proof:

The proof of (i) stems directly from Theorem 1 in Erreygers *et al.* (2012) in the one-dimensional setting. Indeed, $M(\mathbf{H})$ respects (SYM) if, and only if, $M_i(\mathbf{H})$ satisfies (SYM) for all $i = 1, \ldots, n$. The weight $w_i(\cdot)$ is a starshaped function at point $p = 0.5$. Result (ii) is also a consequence of the results of Erreygers *et al.* (2012, p.262) in the one-dimensional case. Indeed, property (MIR) can be obtained if the function $f(\cdot)$ is independent of μ_{ϕ_i}.
■

We then deduce the properties of $I(\mathbf{H})$:[13]

$$I(\mathbf{H}) = \sum_{i=1}^{n} \theta_i \left[\frac{1}{\mu_{\phi_i}} \int_0^1 \left(1 - \alpha_i s(1-p)^{s-1}\right) \phi_i(\mathbf{H}(\mathbf{p}))dp \right]. \qquad \text{(not MIR not SYM)}$$

In order to obtain (MIR) in a multivariate context, the aggregable concentration index $I(\mathbf{H})$ may be generalized, as in Erreygers *et al.* (2012) for the unidimensional cases. Aggregable socioeconomic health inequality indices $I(\mathbf{H}) \in \mathcal{M}$ with $f(\cdot)$ independent of μ_ϕ can be defined, for each $\alpha_i > 0$, as:

$$GC(\mathbf{H}) = \sum_{i=1}^{n} \theta_i \left[\frac{s^{s/(s-1)}}{s-1} \int_0^1 \alpha_i \left(1 - s(1-p)^{s-1}\right) \phi_i(\mathbf{H}(\mathbf{p}))dp \right] \qquad \text{(MIR not SYM)}$$

$$GS(\mathbf{H}) = \sum_{i=1}^{n} \theta_i \left[\int_0^1 \alpha_i \left(p - \frac{1}{2}\right)^{s-1} \phi_i(\mathbf{H}(\mathbf{p}))dp \right], \ s \text{ even.} \qquad \text{(MIR and SYM)}$$

They represent the multivariate counterpart of the generalized extended concentration index and the generalized symmetric index, respectively. For the former, setting $w_i(p, s) = \alpha_i - \alpha_i s(1-p)^{s-1}$ such that $\alpha_i s(1-p)^{s-1} = v_i(p)$ implies the respect of the s-th positional transfer principle, that is $GC(\mathbf{H}) \in \{\mathcal{M} \cap \Xi^s\}_{s \geqslant 2}$. Thus, the ranking of two health distributions $\tilde{\mathbf{H}}$ and $\mathbf{H}$ is given by the sequential dominance of the generalized achievement curves $GAC^s_{\tilde{\mathbf{H}},i}$ over $GAC^s_{\mathbf{H}_i}$.

[13]It is noteworthy that (MIR) would not be respected even if $\mu_{\phi_i} = c$ is a constant. Indeed, if each $\alpha_i > 0$, it is possible to show that $\int_0^1 w_i(p, s)dp = \int_0^1 \left(1 - \alpha_i s(1-p)^{s-1}\right) dp \neq 0$, and in this case (MIR) cannot be fulfilled.

Theorem 5.1 *Under Definition 3.1, for all aggregable socioeconomic health inequality indices $GC(\mathbf{H}) \in \mathcal{M}$ respecting* (MIR) *but not* (SYM) *such that $w_i(p,s) = \alpha_i - \alpha_i s(1-p)^{s-1} = \alpha_i - v_i(p)$ and $s \in \{1,2,3,\ldots\}$, sufficient conditions for $GC(\tilde{\mathbf{H}}) \leqslant GC(\mathbf{H})$ are:*

$$\sum_{i=1}^{l} \theta_i \left[GAC^{s}_{\mathbf{H},i}(p) - GAC^{s}_{\tilde{\mathbf{H}},j}(p) \right] \leqslant 0, \ \forall l \in \{1,\ldots,n\}, \ \forall p \in [0,1],$$

and in addition, if $s \geqslant 4$,

$$\sum_{i=1}^{l} \theta_i \left[GAC^{u}_{\mathbf{H},i}(1) - GAC^{u}_{\tilde{\mathbf{H}},i}(1) \right] \leqslant 0, \ \forall u \in \{3,4,\ldots,s-1\}, \ \forall l \in \{1,\ldots,n\}.$$

Proof:

See the Appendix. ■

As can be seen in the previous result, compared with that of Theorem 4.2, additional dominance conditions appear at the location $p = 1$ whenever $s \geqslant 4$ for the generalized achievement curves.

Alternatively, it is possible to show that $GS(\mathbf{H}) \notin \Xi^s$ because the sth positional transfer principle does not hold for all orders. Indeed, (SYM) requires that $w_i(p,s) = \alpha_i \left(p - \frac{1}{2}\right)^{s-1} = -\alpha_i \left(\frac{1}{2} - p\right)^{s-1} = -w_i(1-p,s)$ if, and only if, s is even for $p \in [0,1]$. By consequence, setting $v_i(p) = w_i(p) = \alpha_i \left(p - \frac{1}{2}\right)^{s-1}$ implies that the dominance conditions of Theorem 5.1 no longer hold.

However, another result is available for inequality lovers. Let us analyze the variations of the generalized symmetric index. First, it is notable that $GS(\mathbf{H})$ reaches its maximum when the individuals whose equivalent income is above the median ($p \geqslant \frac{1}{2}$) are all healthy, $\phi(\mathbf{H}(\mathbf{p})) = 1$, whereas those below the median are all unhealthy $\phi(\mathbf{H}(\mathbf{p})) = 0$. Second, if the reverse situation occurs, then the index will reach its minimum.[14] Third, if all individuals in the population are healthy (or unhealthy), then $GS(\mathbf{H}) = 0$. In other words, the index increases with inequalities for all $p \geqslant \frac{1}{2}$ and decreases with inequalities for all $p \leqslant \frac{1}{2}$. Hence, $GS(\mathbf{H})$ is sensitive to health variations, *i.e.* health transfers, which take place on each side of the median. In the case where $GS(\mathbf{H})$ respects the positional transfer of order 2, for transfers occurring above the median in group i only, then it comes that $v_i^{(1)}(p) \geqslant 0$ for all $p \geqslant \frac{1}{2}$. If $GS(\mathbf{H})$ respects the positional transfer of order 2 for transfers below the median in group i, then $v_i^{(1)}(p) \leqslant 0$ for all $p \leqslant \frac{1}{2}$. As a consequence, setting $w_i(p,s) = v_i(p) = \alpha_i \left(p - \frac{1}{2}\right)^{s-1}$, it is apparent that positional transfers of order 2 ($s = 2$) below the median are not fulfilled, because $v_i^{(1)}(p) > 0$ whenever $\alpha_i > 0$. Hence, maintaining the property of symmetry (SYM) comes at a cost, since transfers to poor individuals or to those below the median intensify socio-economic health inequalities. The normative interpretation of the index $GS(\mathbf{H})$ is that the decision maker behind

[14] If $\alpha_i = 8$ for all $i = 1,\ldots,n$, then the maximal value is $GS(\mathbf{H}) = 1$ and the minimal one is $GS(\mathbf{H}) = -1$.

the veil of ignorance is inclined to performed health transfers to reduce socioeconomic health inequalities for people above the median only. The decision maker is said to be an inequality lover with regard to the lower tail of the distribution of equivalent incomes, in other words, a downward inequality lover.[15] This result is formalized as follows.

Theorem 5.2 *Under vertical equity of Definition 3.1, for all aggregable socioeconomic health inequality indices* $GS(\mathbf{H}) \in \mathcal{M}$ *respecting* (MIR) *and* (SYM) *such that* $w_i(p,s) = v_i(p) = \alpha_i \left(p - \frac{1}{2}\right)^{s-1}$, *sufficient conditions for* $GS(\tilde{\mathbf{H}}) \leqslant GS(\mathbf{H})$ *are:*

$$\sum_{i=1}^{l} \theta_i \left[GAC^s_{\mathbf{H},i}(p) - GAC^s_{\tilde{\mathbf{H}},j}(p) \right] \leqslant 0, \ \forall l \in \{1, \ldots, n\}, \ \forall p \in [0,1], \ s = 2.$$

Proof:

See the Appendix. ■

Theorem 5.2 shows that a sequential dominance criterion exists in order to rank health distributions when the decision maker has to account for mirror and symmetry simultaneously. However, it is notable that the dominance condition is more restrictive than in Theorem 5.1, because above the second-order dominance, some contradictions arise (see the proof) in that the value judgments are limited to the second-order positional transfers only. Moreover, for those transfers, the paradox discussed above emerges, concerning the impossibility of health transfers for people below the median. Since poor people might be affected by exogenous risk factors, it is then impossible to find a compensation principle based on redistributive actions, such as positional transfers. In other words, for all multidimensional generalized concentration indices $GS(\mathbf{H}) \in \mathcal{M}$ consistent with (MIR) and (SYM), there is no relevant dominance criterion with regard to *differences in exogenous risk factors.*

6 Conclusion

In this paper, three main findings are itemized. First, a dominance result is provided for heterogeneous agents, in a so-called sequential dominance test. Second, the class of achievement (and inequality) health indices is reduced to polynomial functions relevant to concentration indices and generalized concentration indices. Third, the properties (MIR) and (SYM) are shown to be consistent with a sub-family of polynomial functions for which the stochastic dominance test is reversed compared with the usual one. Accordingly, either (SYM) or (MIR) may be relaxed to design socioeconomic health inequalities relevant to social planners averse to exogenous risk factors.

[15] See Aaberge (2009) for decision makers that support the upward and downward positional transfer principles in a rank-dependent layout with the related concepts of upward and downward Lorenz dominance.

Future research could be done by following the decomposition approach. Indeed, the class of decomposable indices $I'(\mathbf{H})$ could be investigated in order to perform dominance tests assessing within-group and between-group socioeconomic health inequalities.

7 Appendix: Proof

Theorem 4.1 *Under Definition 3.1, for all health achievement indices* $A(\mathbf{H}) \in \Omega^s$, where $s \in \{1, 2, 3, \ldots\}$, *respecting* (A1) *and* (A2), *the two following statements are equivalent:*

(i) $A(\tilde{\mathbf{H}}) \geqslant A(\mathbf{H})$

(ii) $\sum_{i=1}^{l} \kappa_i \left[GAC^s_{\tilde{\mathbf{H}},j}(p) - GAC^s_{\mathbf{H},i}(p) \right] \geqslant 0,\ \forall l \in \{1, \ldots, n\},\ \forall p \in [0, 1]$.

Proof:

Sufficiency.

Note that:

$$
\begin{aligned}
A(\tilde{\mathbf{H}}) - A(\mathbf{H}) &= \sum_{i=1}^{n} \kappa_i \int_0^1 v_i(p) \left[\phi_i(\tilde{\mathbf{H}}(\mathbf{p})) - \phi_i(\mathbf{H}(\mathbf{p})) \right] dp \\
&= \sum_{i=1}^{n} \kappa_i \int_0^1 v_i(p) \left[\frac{K - \Upsilon_i(\tilde{\mathbf{H}}(\mathbf{p}))\Theta_i'}{K} - \frac{K - \Upsilon_i(\mathbf{H}(\mathbf{p}))\Theta_i'}{K} \right] dp.
\end{aligned}
$$

Since $GAC^1_{\mathbf{H},i}(p) := (K - \Upsilon_i(\mathbf{H}(\mathbf{p}))\Theta_i')/K$,

$$
A(\tilde{\mathbf{H}}) - A(\mathbf{H}) = \sum_{i=1}^{n} \kappa_i \int_0^1 v_i(p) \left[GAC^1_{\tilde{\mathbf{H}},i}(p) - GAC^1_{\mathbf{H},i}(p) \right] dp.
$$

Integrating s times by parts $\int_0^1 v_i(p) GAC^1_{\mathbf{H},i}(p)\, dp$ such that $v_i^{(\ell)}(1) = 0$ for all $\ell \in \{0, 1, 2, \ldots\}$ and $GAC^2_{\mathbf{H},i}(1) = 0$ by construction, we get that:

$$
\int_0^1 v_i(p) GAC^1_{\mathbf{H},i}(p)\, dp = (-1)^{s-1} \int_0^1 v_i^{(s-1)}(p)\, GAC^s_{\mathbf{H},i}(p)\, dp,
$$

and in the same manner,

$$
\int_0^1 v_i(p) GAC^1_{\tilde{\mathbf{H}},i}(p)\, dp = (-1)^{s-1} \int_0^1 v_i^{(s-1)}(p)\, GAC^s_{\tilde{\mathbf{H}},i}(p)\, dp.
$$

Hence,

$$
A(\tilde{\mathbf{H}}) - A(\mathbf{H}) = \sum_{i=1}^{n} \kappa_i (-1)^{s-1} \int_0^1 v_i^{(s-1)}(p) \left[GAC^s_{\tilde{\mathbf{H}},i}(p) - GAC^s_{\mathbf{H},i}(p) \right] dp. \tag{1}
$$

Now we use Abel's Lemma usually employed in the literature of sequential dominance.

Abel's lemma (see *e.g.* Jenkins and Lambert (1993)): Let $\{x_j\}_{j=1}^n$ and $\{y_i\}_{i=1}^n$ be sequences of real numbers. If $x_n \geqslant x_{n-1} \geqslant \ldots \geqslant x_1 \geqslant 0$, then $\sum_{i=j}^{n} y_i \geqslant 0\ \forall j$ is a sufficient condition for $\sum_{i=1}^{n} x_i y_i \geqslant 0$. Contrary to this, if $x_n \leqslant x_{n-1} \leqslant \ldots \leqslant x_1 \leqslant 0$, then $\sum_{i=j}^{n} y_i \geqslant 0 \ \forall j$ is also a sufficient condition for $\sum_{i=1}^{n} x_i y_i \leqslant 0$.

Following Abel's Lemma, $\sum_{i=1}^{l} \kappa_i \left[GAC^s_{\tilde{\mathbf{H}},j}(p) - GAC^s_{\mathbf{H},i}(p)\right] \geqslant 0$, for all $l \in \{1,\ldots,n\}$ is a sufficient condition to get $A(\tilde{\mathbf{H}}) \geqslant A(\mathbf{H})$.

Necessity.
From Lemma 3.1, even if we relax (A3), we can use polynomial functions such as:

$$v_i^{(s-2)}(p) = \begin{cases} \alpha_i (-1)^{s-2} \epsilon & p \leqslant \overline{p} \\ \alpha_i (-1)^{s-2} (\overline{p} + \epsilon - p) & \overline{p} < p \leqslant \overline{p} + \epsilon \\ 0 & p > \overline{p} + \epsilon \\ 0 & \text{if } s = 1 \end{cases},$$

with $\alpha_1 \geqslant \cdots \geqslant \alpha_n \geqslant 0$ by Definition 3.1 of exogenous risk factors (in order to match condition (i) in the case where $s = 2$). By the differentiability assumption included in the set Ω^s, we get for all agent types $i = 1, \ldots, n$:

$$v_i^{(s-1)}(p) = \begin{cases} 0 & p \leqslant \overline{p} \\ \alpha_i (-1)^{s-1} & \overline{p} < p \leqslant \overline{p} + \epsilon \\ 0 & p > \overline{p} + \epsilon \end{cases}. \tag{2}$$

Suppose by contradiction that $\sum_{i=1}^{l} \kappa_i [GAC^s_{\tilde{\mathbf{H}},j}(p) - GAC^s_{\mathbf{H},i}(p)] < 0$ for all $l = 1, \ldots, n$ on an interval $[\overline{p}, \overline{p} + \epsilon]$ for some ϵ close to 0. Thereby, substituting equation (2) into (1), according to $\alpha_1 \geqslant \cdots \geqslant \alpha_n \geqslant 0$ it is easy to show that $A(\tilde{\mathbf{H}}) - A(\mathbf{H}) < 0$ *i.e.* a contradiction. ■

Theorem 4.2 *Under Definition 3.1, for all aggregable socioeconomic health inequality indices* $I(\mathbf{H}) \in \Xi^s$, with $s \in \{1, 2, 3, \ldots\}$, *respecting* (I1) *and* (I2), *the two following statements are equivalent:*

(i) $I(\tilde{\mathbf{H}}) \leqslant I(\mathbf{H})$

(ii) $\sum_{i=1}^{l} \theta_i \left[AC^s_{\mathbf{H},i}(p) - AC^s_{\tilde{\mathbf{H}},j}(p)\right] \leqslant 0, \ \forall l \in \{1, \ldots, n\}, \ \forall p \in [0, 1].$

Proof:
The proof goes along the line of Theorem 4.1:

$$\begin{aligned} I(\tilde{\mathbf{H}}) - I(\mathbf{H}) &= \sum_{i=1}^{n} \theta_i \int_0^1 v_i(p) \left[\left(1 - \frac{\phi_i(\tilde{\mathbf{H}}(\mathbf{p}))}{\mu_{\phi_i(\tilde{\mathbf{H}})}}\right) - \left(1 - \frac{\phi_i(\mathbf{H}(\mathbf{p}))}{\mu_{\phi_i(\mathbf{H})}}\right)\right] dp \\ &= \sum_{i=1}^{n} \theta_i \int_0^1 v_i(p) \left[-\left(\frac{K - \Upsilon_i(\tilde{\mathbf{H}}(\mathbf{p}))\Theta_i'}{K\mu_{\phi_i(\tilde{\mathbf{H}})}}\right) + \left(\frac{K - \Upsilon_i(\mathbf{H}(\mathbf{p}))\Theta_i'}{K\mu_{\phi_i(\mathbf{H})}}\right)\right] dp \\ &= \sum_{i=1}^{n} \theta_i \int_0^1 v_i(p) \left[AC^1_{\mathbf{H},i}(p) - AC^1_{\tilde{\mathbf{H}},i}(p)\right] dp. \end{aligned}$$

Integrating successively by parts the previous expression yields:

$$I(\tilde{\mathbf{H}}) - I(\mathbf{H}) = \sum_{i=1}^{n} \theta_i (-1)^{s-1} \int_0^1 v_i^{(s-1)}(p) \frac{1}{K} \left[AC^s_{\mathbf{H},i}(p) - AC^s_{\tilde{\mathbf{H}},i}(p) \right] dp. \quad (3)$$

The remainder of the proof relies on that of Theorem 4.1. ■

Theorem 5.1 *Under Definition 3.1, for all aggregable socioeconomic health inequality indices $GC(\mathbf{H}) \in \mathcal{M}$ respecting* (MIR) *but not* (SYM) *such that $w_i(p,s) = \alpha_i - \alpha_i s(1-p)^{s-1} = \alpha_i - v_i(p)$ and $s \in \{1,2,3,\ldots\}$, sufficient conditions for $GC(\tilde{\mathbf{H}}) \leqslant GC(\mathbf{H})$ are:*

$$\sum_{i=1}^{l} \theta_i \left[GAC^s_{\mathbf{H},i}(p) - GAC^s_{\tilde{\mathbf{H}},j}(p) \right] \leqslant 0, \; \forall l \in \{1,\ldots,n\}, \; \forall p \in [0,1],$$

and in addition, if $s \geqslant 4$,

$$\sum_{i=1}^{l} \theta_i \left[GAC^u_{\mathbf{H},i}(1) - GAC^u_{\tilde{\mathbf{H}},i}(1) \right] \leqslant 0, \; \forall u \in \{3,4,\ldots,s-1\}, \; \forall l \in \{1,\ldots,n\}.$$

Proof:

Setting for short $w_i(p) := \alpha_i - \alpha_i s(1-p)^{s-1}$ yields:

$$\begin{aligned} GC(\tilde{\mathbf{H}}) - GC(\mathbf{H}) &= \sum_{i=1}^{n} \theta_i \frac{s^{s/(s-1)}}{s-1} \int_0^1 \alpha_i \left(1 - s(1-p)^{s-1}\right) \left[\phi_i(\tilde{\mathbf{H}}(\mathbf{p})) - \phi_i(\mathbf{H}(\mathbf{p})) \right] dp \\ &= \sum_{i=1}^{n} \theta_i \frac{s^{s/(s-1)}}{s-1} \int_0^1 w_i(p) \left[\frac{K - \Upsilon_i(\tilde{\mathbf{H}}(\mathbf{p}))\Theta_i'}{K} - \frac{K - \Upsilon_i(\mathbf{H}(\mathbf{p}))\Theta_i'}{K} \right] dp \\ &= \sum_{i=1}^{n} \theta_i \frac{s^{s/(s-1)}}{s-1} \int_0^1 w_i(p) \left[GAC^1_{\tilde{\mathbf{H}},i}(p) - GAC^1_{\mathbf{H},i}(p) \right] dp. \end{aligned}$$

Integrating by parts $w_i(p) GAC^1_{\mathbf{H},i}(p)$ provides:

$$\begin{aligned} \int_0^1 w_i(p) GAC^1_{\mathbf{H},i}(p) dp &= \left| w_i(p) GAC^2_{\mathbf{H},i}(p) \right|_0^1 - \int_0^1 w^{(1)}(p) GAC^2_{\mathbf{H},i}(p) dp \\ &= w_i^{(0)}(1) GAC^2_{\mathbf{H},i}(1) - \int_0^1 w^{(1)}(p) GAC^2_{\mathbf{H},i}(p) dp. \end{aligned}$$

After integrating successively by parts, since $GAC^s_{\mathbf{H},i}(0) = 0$ for all $s \in \{1,2,3,\ldots\}$, we obtain:

$$\int_0^1 w_i(p) GAC^1_{\mathbf{H},i}(p) dp = \sum_{u=2}^{s} (-1)^{u-2} w_i^{(u-2)}(1) GAC^u_{\mathbf{H},i}(1) + \int_0^1 (-1)^{s-1} w_i(p)^{s-1} GAC^s_{\mathbf{H},i}(p).$$

By definition $GAC^2_{\mathbf{H},i}(1) = 1$ for all $i = 1,\ldots,n$, hence:

$$\begin{aligned} GC(\tilde{\mathbf{H}}) - GC(\mathbf{H}) = &\sum_{i=1}^{n} \theta_i \frac{s^{s/(s-1)}}{s-1} \int_0^1 (-1)^{s-1} w_i(p)^{s-1} \left[GAC^s_{\tilde{\mathbf{H}},i}(p) - GAC^s_{\mathbf{H},i}(p) \right] dp \\ &+ \sum_{u=3}^{s} \sum_{i=1}^{n} \theta_i \frac{s^{s/(s-1)}}{s-1} (-1)^{u-2} w_i^{(u-2)}(1) \left[GAC^u_{\tilde{\mathbf{H}},i}(1) - GAC^u_{\mathbf{H},i}(1) \right]. \end{aligned}$$

By Definition 3.1, $\alpha_1 \geqslant \cdots \geqslant \alpha_n$ ensures vertical equity. By virtue of horizontal equity, we have $(-1)^{s-1}w_1(p)^{s-1} \leqslant \cdots \leqslant (-1)^{s-1}w_n(p)^{s-1} \leqslant 0$ for all $p \in [0,1]$. By Abel's lemma, a sufficient condition for the negativity of the first sum of the equation above is $\sum_{i=1}^{l} \theta_i \big[GAC^s_{\tilde{\mathbf{H}},i}(p) - GAC^s_{\mathbf{H},i}(p)\big] \geqslant 0$ for all $l \in \{1,\ldots,n\}$. Also, for the negativity of the double sum above, Abel's lemma yields $\sum_{i=1}^{l} \theta_i \big[GAC^u_{\tilde{\mathbf{H}},i}(1) - GAC^u_{\mathbf{H},i}(1)\big] \geqslant 0$ for all $u \in \{3,4,\ldots,s\}$ and for all $l \in \{1,\ldots,n\}$. Note that, for $s=3$, the condition $\sum_{i=1}^{l} \theta_i \big[GAC^u_{\tilde{\mathbf{H}},i}(1) - GAC^u_{\mathbf{H},i}(1)\big] \geqslant 0$ does not have to be checked since it is included in $\sum_{i=1}^{l} \theta_i \big[GAC^s_{\tilde{\mathbf{H}},i}(p) - GAC^s_{\mathbf{H},i}(p)\big] \geqslant 0$ where $p \in [0,1]$. ■

Theorem 5.2 *Under vertical equity of Definition 3.1, for all aggregable socioeconomic health inequality indices $GS(\mathbf{H}) \in \mathcal{M}$ respecting* (MIR) *and* (SYM) *such that $w_i(p,s) = v_i(p) = \alpha_i \left(p - \frac{1}{2}\right)^{s-1}$, sufficient conditions for $GS(\tilde{\mathbf{H}}) \leqslant GS(\mathbf{H})$ are:*

$$\sum_{i=1}^{l} \theta_i \left[GAC^s_{\mathbf{H},i}(p) - GAC^s_{\tilde{\mathbf{H}},j}(p)\right] \leqslant 0,\ \forall l \in \{1,\ldots,n\},\ \forall p \in [0,1],\ s=2.$$

Proof:

Setting $v_i(p) = \alpha_i \left(p - \frac{1}{2}\right)^{s-1}$, we get that:

$$\begin{aligned}
GS(\tilde{\mathbf{H}}) - GS(\mathbf{H}) &= \sum_{i=1}^{n} \theta_i \int_0^1 v_i(p) \left[\phi_i(\tilde{\mathbf{H}}(\mathbf{p})) - \phi_i(\mathbf{H}(\mathbf{p}))\right] dp \\
&= \sum_{i=1}^{n} \theta_i \int_0^1 v_i(p) \left[\frac{K - \Upsilon_i(\tilde{\mathbf{H}}(\mathbf{p}))\Theta_i'}{K} - \frac{K - \Upsilon_i(\mathbf{H}(\mathbf{p}))\Theta_i'}{K}\right] dp. \\
&= \sum_{i=1}^{n} \theta_i \int_0^1 v_i(p) \big[GAC^1_{\tilde{\mathbf{H}},i}(p) - GAC^1_{\mathbf{H},i}(p)\big] dp.
\end{aligned}$$

As in Theorem 5.1, integrating successively by parts, we obtain:

$$\int_0^1 v_i(p) GAC^1_{\mathbf{H},i}(p) dp = \sum_{u=2}^{s} (-1)^{u-2} v_i^{(u-2)}(1) GAC^u_{\mathbf{H},i}(1) + \int_0^1 (-1)^{s-1} v_i(p)^{s-1} GAC^s_{\mathbf{H},i}(p).$$

This entails:

$$\begin{aligned}
GS(\tilde{\mathbf{H}}) - GS(\mathbf{H}) = &\sum_{i=1}^{n} \theta_i (-1)^{s-1} \int_0^1 v_i^{(s-1)}(p) \big[GAC^s_{\tilde{\mathbf{H}},i}(p) - GAC^s_{\mathbf{H},i}(p)\big] dp \qquad (4) \\
&+ \sum_{u=3}^{s} \sum_{i=1}^{n} \theta_i (-1)^{u-2} v_i^{(u-2)}(1) \big[GAC^u_{\tilde{\mathbf{H}},i}(1) - GAC^u_{\mathbf{H},i}(1)\big]. \qquad (5)
\end{aligned}$$

Note that for $1 \leqslant k \leqslant s-1$ and $s \in \{2,4,6,\ldots\}$ for the respect of (MIR) and (SYM),

$$v_i^{(k)}(p) = \left[\prod_{r=1}^{k} (s-r)\right] \left(p - \frac{1}{2}\right)^{s-k-1}.$$

Case 1: $p \geqslant \frac{1}{2}$. Clearly, $v_i^{(k)}(p) \geqslant 0$.
Case 2: $p \leqslant \frac{1}{2}$. Since s is even, then the $(s-1)$-order of the derivative of $v_i(\cdot)$ is odd. Then k is odd, implying that $s-k-1$ is even. Then, $v_i^{(k)}(p) \geqslant 0$ for all $s \in \{2, 4, \ldots\}$. By Definition 3.1, $\alpha_1 \geqslant \cdots \geqslant \alpha_n$ to match vertical equity. Hence $v_1(p)^{(s-1)} \geqslant \cdots \geqslant v_n(p)^{(s-1)} \geqslant 0$ for all $p \in [0,1]$, and so $(-1)^{s-1}v_1(p)^{(s-1)} \leqslant \cdots \leqslant (-1)^{s-1}v_n(p)^{(s-1)} \leqslant 0$. Following Abel's Lemma, $\sum_{i=1}^{l} \theta_i \left[GAC^s_{\tilde{\mathbf{H}},j}(p) - GAC^s_{\mathbf{H},i}(p)\right] \geqslant 0$, for all $l \in \{1, \ldots, n\}$ is a sufficient condition to get the negativity of the sum in (4). For the negativity of the sum in (5), the sufficient condition is $\sum_{i=1}^{l} \theta_i \left[GAC^u_{\tilde{\mathbf{H}},j}(1) - GAC^u_{\mathbf{H},i}(1)\right] \geqslant 0$, for all $l \in \{1, \ldots, n\}$ and for $u = 3$ only. Indeed, for u being even such that $u = 4 = s$, we get $(-1)^{u-2}v_i^{(u-2)}(1) \geqslant 0$, which is a contradiction of Abel's lemma used for (4). However, if $u = 3 = s$, in this case s is odd, so that (MIR) and (SYM) does not hold. Hence, the value of s is reduced to $s = 2$, so that the condition (5) becomes irrelevant. ■

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